**Math 253:** Sequences and Series **(4-0-4)** 05/28/20

**Catalog Description:** Indeterminate forms and improper integrals. Infinite sequences and series, convergence, power series. Taylor series and applications.

**Course Objectives:** After completing this course, students will be able to

1. Recognize and use sequences.
2. Recognize, classify, and determine the convergence of numerical series.
3. Recognize and determine the convergence of power series and Taylor series.
4. Determine the Fourier Series of a function.
5. Communicate mathematical ideas using correct and appropriate notation.

## Learning Outcomes and Performance Criteria

1. Demonstrate an understanding of sequences.

   Core Criteria:

   (a) Determine if an expression is a sequence.
   (b) Determine the closed form of a sequence and expand the closed form of a sequence.
   (c) Determine and justify if a sequence converges or diverges.
   (d) Determine the limit of a convergent sequence.
   (e) Determine if a sequence is bounded and find a bound.
   (f) Determine if a sequence is increasing, decreasing or neither.

   Additional Criteria:

   (a) Evaluate a limit using L'Hopital's rule.

2. Demonstrate an understanding of numerical series.

   Core Criteria:

   (a) Give the closed form of a series. Expand the closed form of a series.
   (b) Give partial sums for a series, and find the general partial sum of a series.
   (c) Find the sum of a series as the limit of the sequence of partial sums.
   (d) Determine if a series is geometric and if it converges find the sum.
   (e) Use the divergence test to determine if a series diverges.
   (f) Use linearity properties to compute sums of series.
   (g) Use the integral test, comparison test, limit comparison test to determine if a positive series converges.
   (h) Use the alternating series test, ratio test, and root test to determine if a series converges.
   (i) Determine if a series converges absolutely, conditionally, or not at all.
   (j) Select and apply an appropriate convergence test.

Additional Criteria:

(a) Use the comparison test to determine if an improper integral converges.

3. Demonstrate an understanding of power series.

   Core Criteria:

   (a) Determine a Taylor polynomial and remainder term of a function.
   (b) Determine error bounds for the remainder of a Taylor polynomial.
   (c) Find the Taylor series of $e^x, \sin(x), \cos(x), \ln(1+x),$ and $\dfrac{1}{1+x}$.
   (d) Find the power series of other functions by manipulation (algebraic, substitution, differentiation, integration) of known power series.
   (e) Determine the radius of convergence of a series, and determine the interval convergence (including endpoints).

   Additional Criteria:

   (a) Evaluate or approximate a definite integral of a function by term by term integration of its power series.
   (b) Find the series solution of a differential equation.

4. Demonstrate an understanding of Fourier Series.

   Core Criteria:

   (a) Determine the Fourier series of a periodic function.
   (b) Create an odd or even extension of a function.
   (c) Determine the Fourier Sine or Cosine series of a function defined on the interval $[0, L]$.

   Additional Criteria:

   (a) Determine the complex Fourier series of a periodic function.
   (b) Calculate the discrete Fourier transform of a finite sequence.

5. Demonstrate an understanding of miscellaneous topics.

   Additional Criteria:

   (a) Define, integrate, and differentiate hyperbolic trigonometric functions.
   (b) Find the Taylor series of $\sinh(x), \cosh(x)$.
   (c) Solve differential equations with Fourier series.