Math 254: Multivariable and Vector Calculus **(4-0-4)** 5/29/20

Catalog Description: Vectors, vector functions, and curves in two and three dimensions. Surfaces, partial derivatives, gradients, and directional derivatives. Multiple integrals using rectangular and other coordinate systems. Physical and geometric applications.

Course Objectives: After completing this course, students will be able to

1. Compute vectors in two- and three-space, lines and planes in three-space.
2. Find vector-valued functions of one variable (parametric curves) and compute their derivatives.
3. Find functions of two and three variables and compute their derivatives.
4. Perform multi-dimensional integration.
5. Communicate mathematical ideas using correct and appropriate notation.

Learning Outcomes and Performance Criteria

1. Understand vectors in two- and three-space, lines and planes in three-space, and be able to perform associated computations.

   Core Criteria:

   (a) Find the vector from one point to another.
   (b) Perform algebraic operations with vectors.
   (c) Find the norm of a vector. Find a vector satisfying given direction and magnitude criteria.
   (d) Find the dot product of two vectors. Determine whether two vectors are orthogonal (perpendicular). Determine the angle between two vectors using the dot product.
   (e) Find the cross product of two vectors in 3-space. Know that the cross product of two vectors in 3-space is *a vector* that is orthogonal to both original vectors.
   (f) Tell whether a given operation with scalars and/or vectors is defined.
   (g) Find the parametric equations for a line
      i. through a given point and parallel to a given vector,
      ii. through two given points in 2 or 3-space.
   (h) Draw the projection of one drawn vector on another drawn vector.
   (i) Draw the components of a vector $\mathbf{v}$ that are parallel and perpendicular to a vector $\mathbf{b}$. Find the components algebraically.
   (j) Determine whether two planes are parallel, perpendicular, or neither. Determine whether a line and a plane are parallel, perpendicular, or neither.
   (k) Draw the scalar multiple of a drawn vector, or the sum or difference of two drawn vectors. Illustrate both the parallelogram method and tip-to-tail method for adding two vectors.

(l) Find the equation of a plane, given

i. a point on the plane and a normal vector to the plane,

ii. three points on the plane.

Additional Criteria:

(a) Find the point of intersection of a line and a plane.

2. Understand vector-valued functions of one variable (parametric curves) and their derivatives, perform associated computations, and apply understanding and computations to solve problems.

Core Criteria:

(a) Find the rectangular equation of the path for parametric motion in two dimensions and identify its "shape" (line, circle, ellipse, etc.).

(b) Find velocity, speed and acceleration for parametric motion in two or three dimensions.

(c) Solve an initial value problem for parametric motion in two or three dimensions.

(d) Apply initial value problem methods to solve projectile motion problems.

Additional Criteria:

(a) Determine when and where a particle reaches maximum or minimum speed, and find that speed.

(b) Given the path and direction of motion of a particle and information about whether it is speeding up, slowing down, or moving at a constant speed at a point, sketch possible velocity and acceleration vectors at that point. Sketch possible tangential and normal components of the acceleration at that point.

(c) Given the velocity and acceleration of a particle, determine whether the particle is (a) speeding up, slowing down or moving at constant speed and (b) whether the path of the particle is straight or curved.

(d) Find displacement and distance traveled for parametric motion in two or three dimensions.

(e) Find the curvature of a path at a given point for a planar curve.

(f) Find the curvature of a path at a given point for a space curve.

(g) Find the tangential and normal *vector* components of the acceleration vector. Find the tangential and normal *scalar* components of the acceleration vector. Write the acceleration vector at some time in the form $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$.

3. Understand single-valued functions of two or three variables and their derivatives, perform associated computations, and apply understanding and computations to solve problems.

Core Criteria:

(a) Match a level curve plot of a function of two variables with its three-dimensional plot.

(b) Find and interpret (give location, direction, change in dependent variable per unit of change in independent variable) partial derivatives of a function at a point.

(c) Apply the chain rule to compute the derivative of a composition of functions.

(d) Find vectors and a plane tangent to a surface at a point.

(e) Find and interpret (location, direction, change in dependent variable per unit of change in independent variables) directional derivatives of a function at a point.

(f) Compute the gradient of a function.

(g) Determine the direction in which a function has the greatest rate of increase or decrease at a point, and give that rate. Determine the directions from a point in which a function remains constant.

(h) Find local and/or absolute minima and maxima of a function of one or two variables. (This includes both the function values *and* where they occur.)

Additional Criteria:

(a) Given a level curve plot of a function of two variables, determine the locations and approximate values of absolute maxima and minima on a closed region.

(b) Use calculus to find absolute maxima and minima of a function of two variables on a closed region.

(c) Determine the average rate of change of a function of two or three variables from one point to another.

4. Understand multiple and iterated integrals, perform associated computations, and apply understanding and computations to solve problems.

Core Criteria:

(a) Evaluate a simple double or triple iterated integral "by hand".

(b) Set up and compute a double integral over a given region.

(c) Transform double integrals between rectangular and polar coordinates.

(d) Set up and compute a polar double integral over a given region.

Additional Criteria:

(a) Set up and compute a triple integral over a given volume.

(b) Find the mass of a plate or solid object with variable density. Set up an expression for finding the centroid of an object.