Math 421: Applied Partial Differential Equations I **(4-0-4)** 05/01/20

Catalog Description: The first course in a three term sequence in applied partial differential equations. Modelling physical systems using differential equations, classifying differential equations and introduction to the methods of solving partial differential equations (separation of variables, Fourier series, transform methods).

Course Objectives: After completing this course, students will be able to

1. Formulate a model for systems for partial differential equations including but not limited to the Heat, Wave, Laplace, and Poisson equations in one and two dimensions.
2. Transform an equation or operator from one geometry to another.
3. Classify systems of partial differential equations.
4. Solve partial differential equations.
5. Graph analytical solutions to the following set of example problems.

Learning Outcomes and Performance Criteria

1. Formulate boundary value problems using appropriate coordinates.

   Core Criteria:

   (a) Formulate the proper equation and conditions (initial and boundary) for a cylindrical, homogeneous, isotropic rod with flat ends and given some initial temperature distribution and boundary conditions.

   (b) Formulate the proper equation and conditions (initial and boundary) for a string with uniform density and tension and a given some initial configuration and boundary conditions.

   (c) Formulate the proper equation and boundary conditions for the steady-state temperature of a thin circular disc given boundary conditions.

   (d) Formulate the proper equation and conditions (initial and boundary) for a circular membrane with uniform density and tension given some initial configuration and boundary conditions.

   (e) Choose the appropriate coordinate system for a given geometry.

   Additional Criteria:

   (a) Formulate the proper equation and conditions (initial and boundary) for other domains given some initial temperature distribution or initial configuration and boundary conditions.

   (b) Given the proper equation and conditions for some physical system, reformulate the equation and/or conditions given some change(s) to the system.

2. Transform an equation or operator from one geometry to another.

   Core Criteria:

(a) Transform the 2D heat equation from Cartesian to Polar coordinates.
(b) Transform the 2D wave equation from Cartesian to Polar coordinates.

Additional Criteria:

(a) Transform the 3D heat equation from Cartesian to Spherical coordinates.
(b) Transform a 2D Poisson Equation from Cartesian to Polar Coordinates.

3. Classify differential equations.

   Core Criteria:

   (a) Given a differential equation, determine if the equation is linear or non-linear.
   (b) Given a differential equation, determine if the equation is parabolic, hyperbolic, or elliptic.
   (c) Given a differential equation, determine if the equation is homogeneous or non-homogenous.

4. Solve one and two dimensional partial differential equations of the heat, wave, and Laplace type in both cartesian and polar coordinates.

   Core Criteria:

   (a) Use separation of variables to transform a partial differential equation, in a given geometry, into a set of equivalent ordinary differential equations.
   (b) Properly assign initial conditions and boundary values to their respective ordinary differential equations.
   (c) Use eigenvalues, eigenfunctions and initial-value solution techniques to solve the set of ordinary differential equations that result from a partial differential equation. This includes Fourier Series as appropriate.
   (d) Use superposition properly to construct a partial differential equation solution.
   (e) Use the series form of a forcing function to equate like terms and find unknown constants for non-homogeneous equations.
   (f) Given a partial differential equation, show via substitution and differentiation that a solution solves (or does not solve) an equation of interest.

   Additional Criteria:

   (a) Given the proper classification, explain the most likely solution method.
   (b) Use the method of characteristics to solve partial differential equations.
   (c) Use Fourier Transforms to solve partial differential equations.

5. Graph analytical solutions to the following set of example problems.

   Core Criteria:

   (a) 1D wave equation.
   (b) 1D heat equation
   (c) 2D Laplace equation in cartesian and polar coordinates.

And one from the following list.

(a) 2D wave equation in cartesian and polar coordinates.

(b) 2D heat equation in cartesian and polar coordinates.

(c) 2D Heat or Laplace equation in an annulus or wedge (pie) shaped region.